

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



Level 2 Certificate in Further Mathematics  
January 2013

# Further Mathematics

**8360/2**

## Level 2

**Paper 2 Calculator**

**Tuesday 29 January 2013 1.30 pm to 3.30 pm**

<p><b>For this paper you must have:</b></p> <ul style="list-style-type: none"> <li>• a calculator</li> <li>• mathematical instruments.</li> </ul>	
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### Time allowed

- 2 hours

### Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16 – 17	
18 – 19	
20 – 21	
22 – 23	
<b>TOTAL</b>	

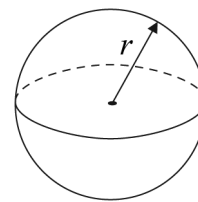


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## Formulae Sheet

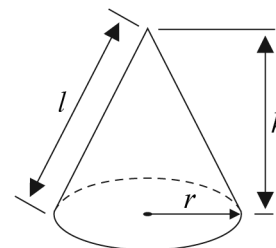
$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Surface area of sphere} = 4\pi r^2$$



$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Curved surface area of cone} = \pi r l$$



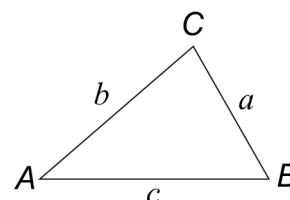
In any triangle  $ABC$

$$\text{Area of triangle} = \frac{1}{2}ab \sin C$$

$$\text{Sine rule} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule} \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



### The Quadratic Equation

The solutions of  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

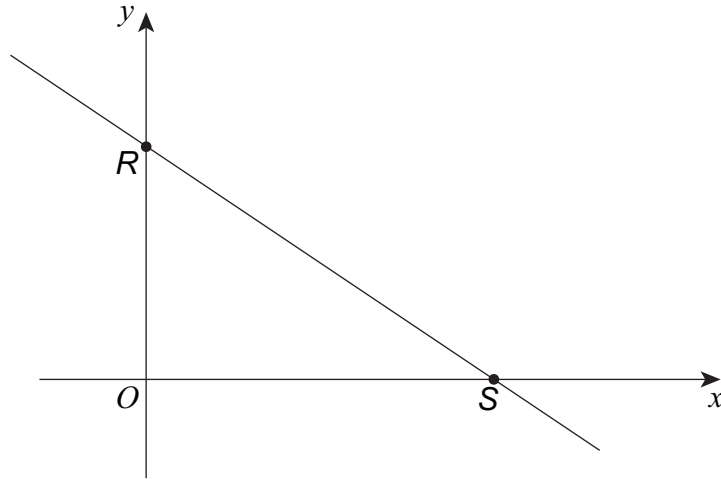
### Trigonometric Identities

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \quad \sin^2 \theta + \cos^2 \theta \equiv 1$$



Answer **all** questions in the spaces provided.

- 1** A sketch of  $2x + 3y = 12$  is shown.



- 1 (a)** Work out the coordinates of  $R$ .

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Answer ( ..... , ..... ) (1 mark)

- 1 (b)** Work out the coordinates of the midpoint of  $RS$ .

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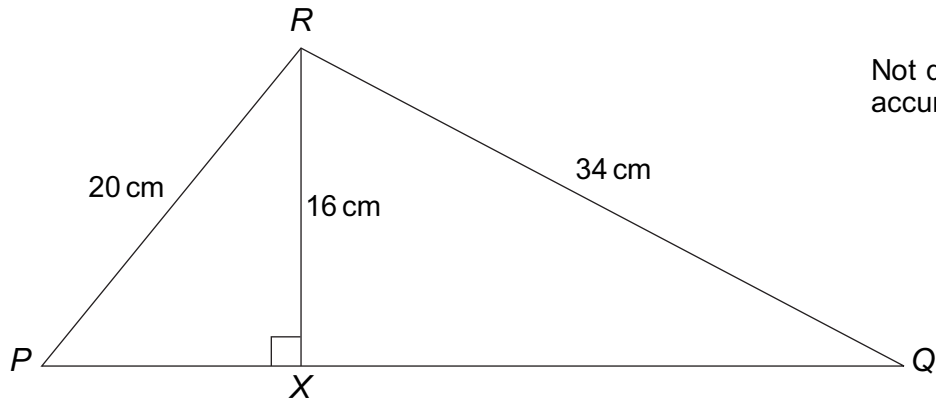
Answer ( ..... , ..... ) (2 marks)

Turn over ►



2

In triangle  $PQR$ ,  $X$  is a point on  $PQ$ .  
 $RX$  is perpendicular to  $PQ$ .



Work out the ratio  $PX:XQ$   
 Give your answer in its simplest form.

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Answer ..... : .....

(4 marks)



3 Solve  $5d - 3 > d + 17$

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Answer ..... (2 marks)

4 Match each statement with an equation.  
You will **not** use all of the equations.

One has been done for you.

A curve passing through (0, 0)

A curve passing through (1, 0)

A circle centre (2, -1)

A circle passing through (3, 1)

$x^2 + y^2 = 10$

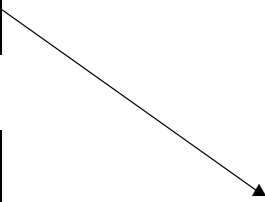
$(x + 2)^2 + (y - 1)^2 = 1$

$y = x^3$

$y = x^3 + x - 2$

$(x - 2)^2 + (y + 1)^2 = 1$

$y = x^2 - 2$



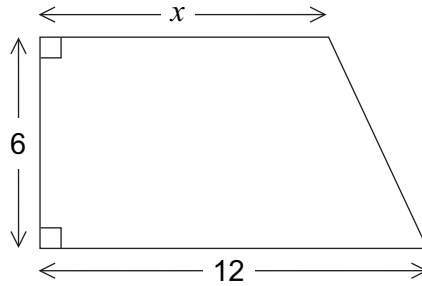
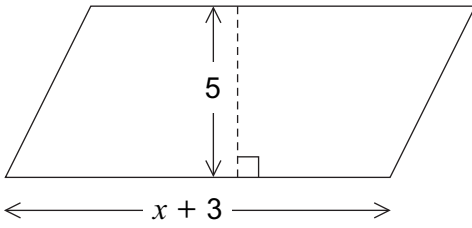
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(3 marks)

Turn over ►



5 A parallelogram and a trapezium are shown.  
All lengths are in centimetres.



Not drawn  
accurately

The area of the parallelogram is equal to the area of the trapezium.

Work out the value of  $x$ .

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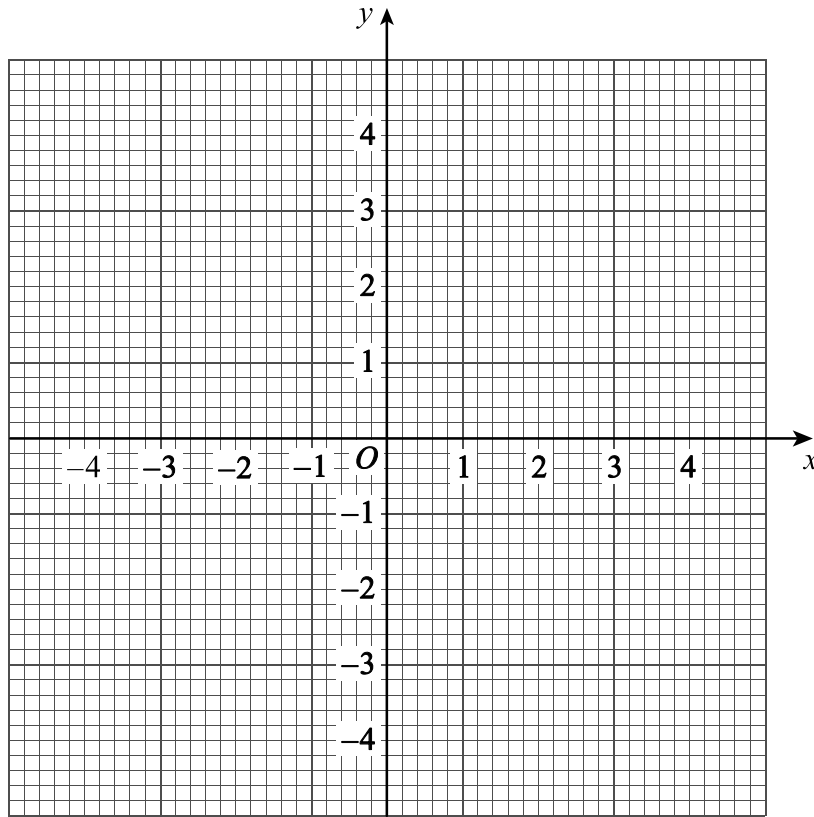
$x =$  ..... cm (4 marks)



6 A function  $f(x)$  is defined as

$$\begin{aligned} f(x) &= 4 && x < -2 \\ &= x^2 && -2 \leq x \leq 2 \\ &= 12 - 4x && x > 2 \end{aligned}$$

6 (a) Draw the graph of  $y = f(x)$  for  $-4 \leq x \leq 4$



(3 marks)

6 (b) Use your graph to write down **how many** solutions there are to  $f(x) = 3$

Answer ..... (1 mark)

6 (c) Solve  $f(x) = -10$

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$x =$  ..... (2 marks)



7 Here are the first four terms of a sequence.

$4a$        $9a$        $14a$        $19a$

The  $n$ th term of the sequence is  $\frac{10n - 2}{3}$

Work out the value of  $a$ .

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$a =$  ..... (2 marks)

8 (a) Factorise fully  $5m^2 - 20p^2$

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Answer ..... (3 marks)

8 (b) You are given that  $p = 15$  and  $5m^2 - 20p^2 = 0$

Using your answer to part (a), or otherwise, work out the values of  $m$ .

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Answer ..... (2 marks)





9 (a) Expand  $(x + m)(x + n)$

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Answer ..... (1 mark)

9 (b)  $x^2 + qx + r \equiv (x + m)(x + n)$

Use your answer to part (a) to write  $q$  and  $r$  in terms of  $m$  and  $n$ .

$q =$  .....

$r =$  ..... (2 marks)

9 (c)  $r$  is an odd integer.

Use your answer to part (b) to explain why  $q$  is an even integer.

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(2 marks)

Turn over ►



**10**  $S = \frac{a}{1-r}$

**10 (a)** Show that  $r = \frac{S-a}{S}$

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(3 marks)

**10 (b)** Work out the value of  $r$  when  $S = 10a$

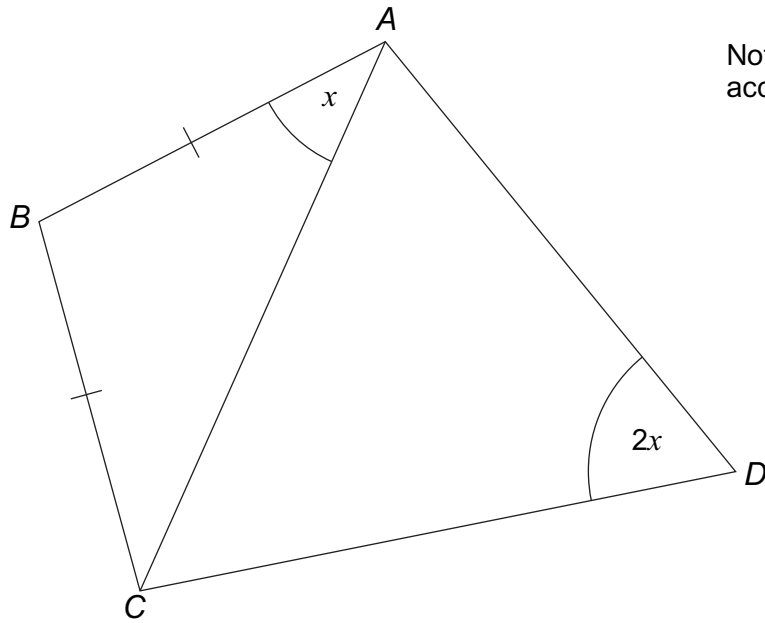
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$r =$  ..... (2 marks)



11

In the diagram,  $AB = BC$



Not drawn  
accurately

Prove that  $ABCD$  is a cyclic quadrilateral.  
Give reasons for any statements you make.

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(3 marks)

Turn over ►



**12**       $f(x) = \sin x$        $180^\circ \leq x \leq 360^\circ$   
          $g(x) = \cos x$        $0^\circ \leq x \leq \theta$

**12 (a)**      Calculate the value of  $f(210^\circ)$ .

Answer ..... (1 mark)

**12 (b)**      Complete this inequality for the range of  $f(x)$ .

Answer .....  $\leq f(x) \leq$  ..... (2 marks)

**12 (c)**      You are given that       $0 \leq g(x) \leq 1$

Work out the value of  $\theta$ .

$\theta =$  ..... degrees (1 mark)



13 (a) Show that  $\frac{4}{x} + \frac{2}{x-1}$  simplifies to  $\frac{6x-4}{x(x-1)}$

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(2 marks)

13 (b) Hence, or otherwise, solve  $\frac{4}{x} + \frac{2}{x-1} = 3$

Give your solutions to 3 significant figures.

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Answer ..... (5 marks)



- 14** The value of  $x$  is 50% **more** than the value of  $t$ .  
The value of  $y$  is 10% **less** than the value of  $w$ .

$$x = y$$

Work out  $\frac{t}{w}$

Give your answer as a decimal.

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$$\frac{t}{w} = \dots\dots\dots (4 \text{ marks})$$

- 15** Describe fully the **single** transformation represented by the matrix  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(3 marks)



16

$$y = (x^3 - 1)^2 + (\sqrt{x})^8$$

Work out  $\frac{dy}{dx}$ .

$$\frac{dy}{dx} = \dots\dots\dots (5 \text{ marks})$$

**Turn over for the next question**

12
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**Turn over** ►



17

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$  represents a reflection in the  $y$ -axis.

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  represents a reflection in the line  $y = x$

Work out the matrix that represents a reflection in the  $y$ -axis followed by a reflection in the line  $y = x$

Answer  $\left( \begin{array}{cc} \dots & \dots \\ \dots & \dots \end{array} \right)$

(2 marks)





**18** Express  $1 - \tan \theta \sin \theta \cos \theta$  in terms of  $\cos \theta$ .

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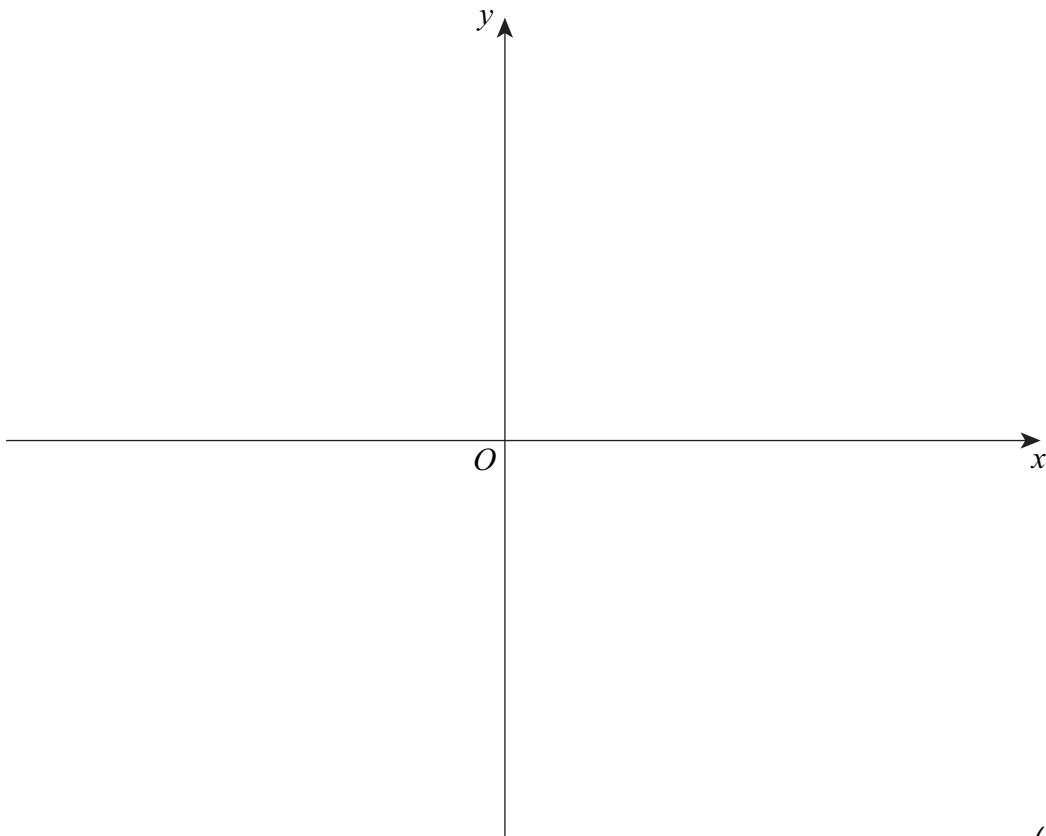
Answer ..... (3 marks)

**19** A cubic function  $f(x)$  has domain  $-4 \leq x \leq 4$

The curve  $y = f(x)$

- has a minimum point at  $(-2, 0)$
- has a maximum point at  $(1, 4)$
- meets the  $x$ -axis at  $(4, 0)$ .

Sketch the graph of  $y = f(x)$  on these axes.  
Label any points where the graph meets the  $x$ -axis.



(4 marks)

Turn over ►





21

Work out the equation of the normal to the curve  $y = x^2 + 4x + 5$  at the point where  $x = -3$

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Answer ..... (5 marks)

22

$f(x) = x^3 + ax^2 + bx + 24$  for all values of  $x$ .

Two of the factors of  $f(x)$  are  $(x - 2)$  and  $(x + 3)$ .

Work out the values of  $a$  and  $b$ .

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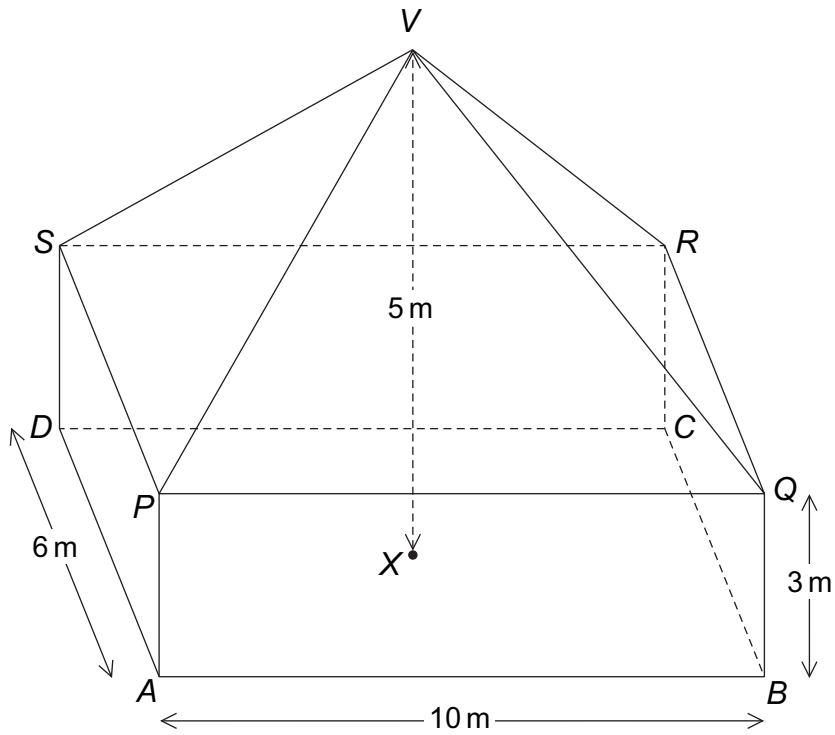
$a =$  .....       $b =$  ..... (5 marks)

Turn over ►



23

The diagram shows a cuboid  $ABCDPQRS$  and a pyramid  $PQRSV$ .  
 $V$  is directly above the centre,  $X$ , of  $ABCD$ .



The total height,  $VX$ , is 5 metres.



**23 (a)** Work out the angle between the line  $VA$  and the plane  $ABCD$ .

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Answer ..... degrees (4 marks)

**23 (b)** Work out the angle between the planes  $VQR$  and  $PQRS$ .

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Answer ..... degrees (2 marks)



24 Solve  $3 \cos^2 \theta - 1 = 0$  for  $0^\circ \leq \theta \leq 180^\circ$

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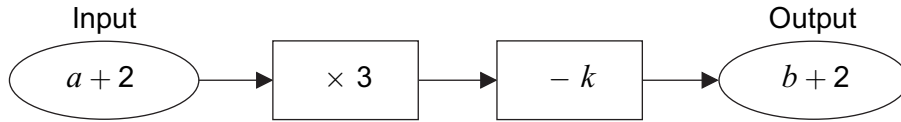
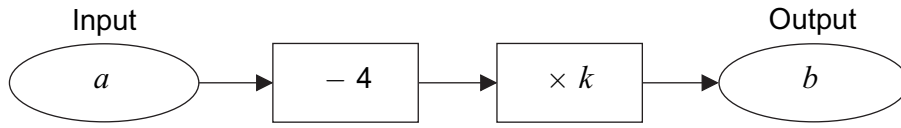
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Answer ..... (4 marks)



25

Here are two number machines.



Work out  $a$  in terms of  $k$ .

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$a =$  ..... (6 marks)

**END OF QUESTIONS**

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ANSWER IN THE SPACES PROVIDED**

