

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



General Certificate of Secondary Education
Higher Tier
November 2012

Mathematics

43603H

Unit 3

Monday 12 November 2012 9.00 am to 10.30 am

H

<p>For this paper you must have:</p> <ul style="list-style-type: none"> • a calculator • mathematical instruments. 	
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Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book.
- If your calculator does not have a π button, take the value of π to be 3.14 unless another value is given in the question.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- The quality of your written communication is specifically assessed in Questions 3 and 16. These questions are indicated with an asterisk (*).
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16 – 17	
18 – 19	
20 – 21	
22 – 23	
TOTAL	



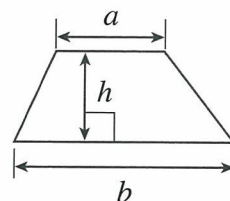
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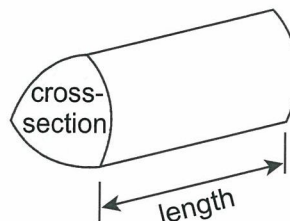
43603H

Formulae Sheet: Higher Tier

Area of trapezium = $\frac{1}{2}(a+b)h$

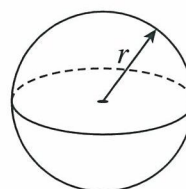


Volume of prism = area of cross-section \times length



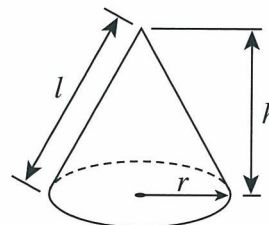
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

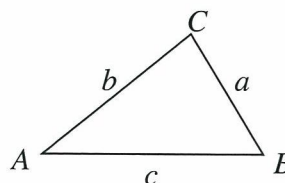


In any triangle ABC

Area of triangle = $\frac{1}{2}ab \sin C$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$



The Quadratic Equation

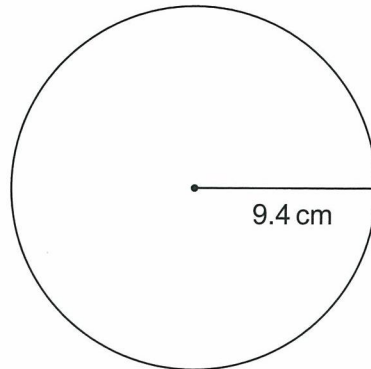
The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Answer **all** questions in the spaces provided.

- 1 A circle has radius 9.4 cm.



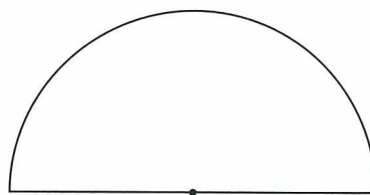
Not drawn accurately

- 1 (a) Work out the circumference of the circle.

$$C = 2\pi r = 2\pi(9.4) = 59.1 \text{ cm (3 s.f.)}$$

Answer 59.1 cm (3 s.f.) (2 marks)

- 1 (b) A semicircle has radius 9.4 cm.



Not drawn accurately

Use your answer to part (a) to work out the perimeter of the semicircle.

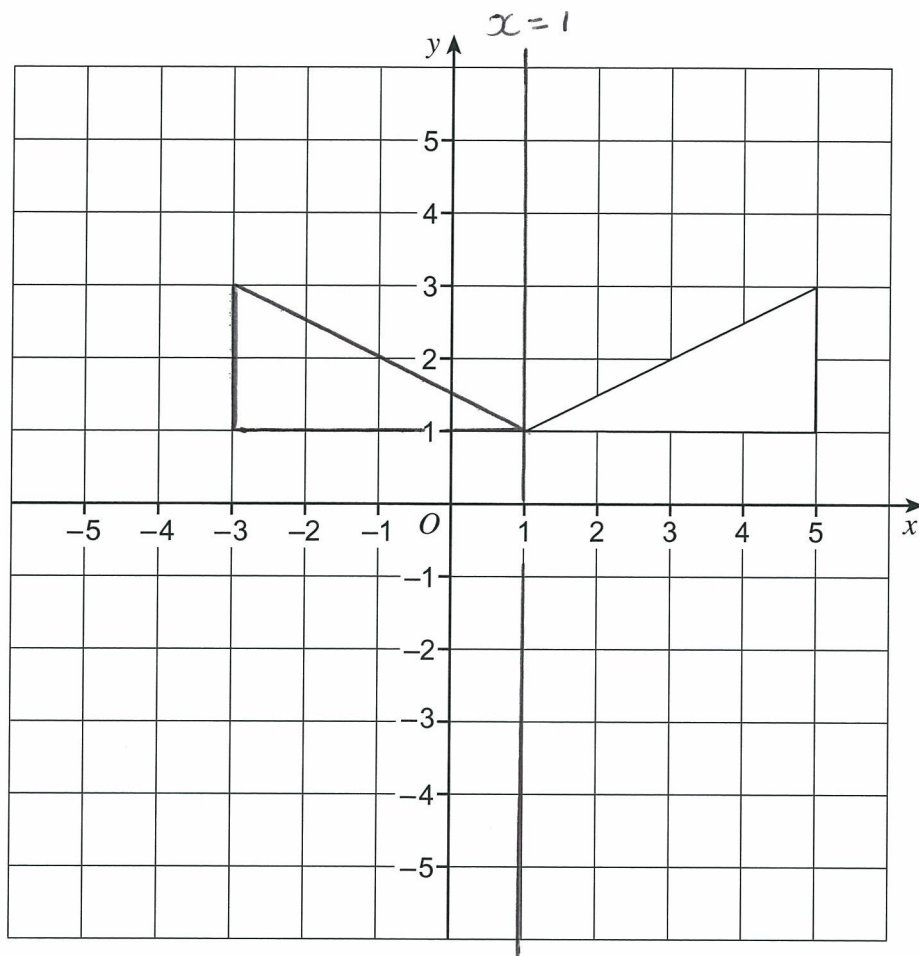
$$\text{Perimeter} = \frac{2\pi r}{2} + 2r = \pi r + 2r = \pi(9.4) + 2(9.4) = 48.3 \text{ cm (3 s.f.)}$$

Answer 48.3 cm (3 s.f.) (2 marks)

Turn over ►



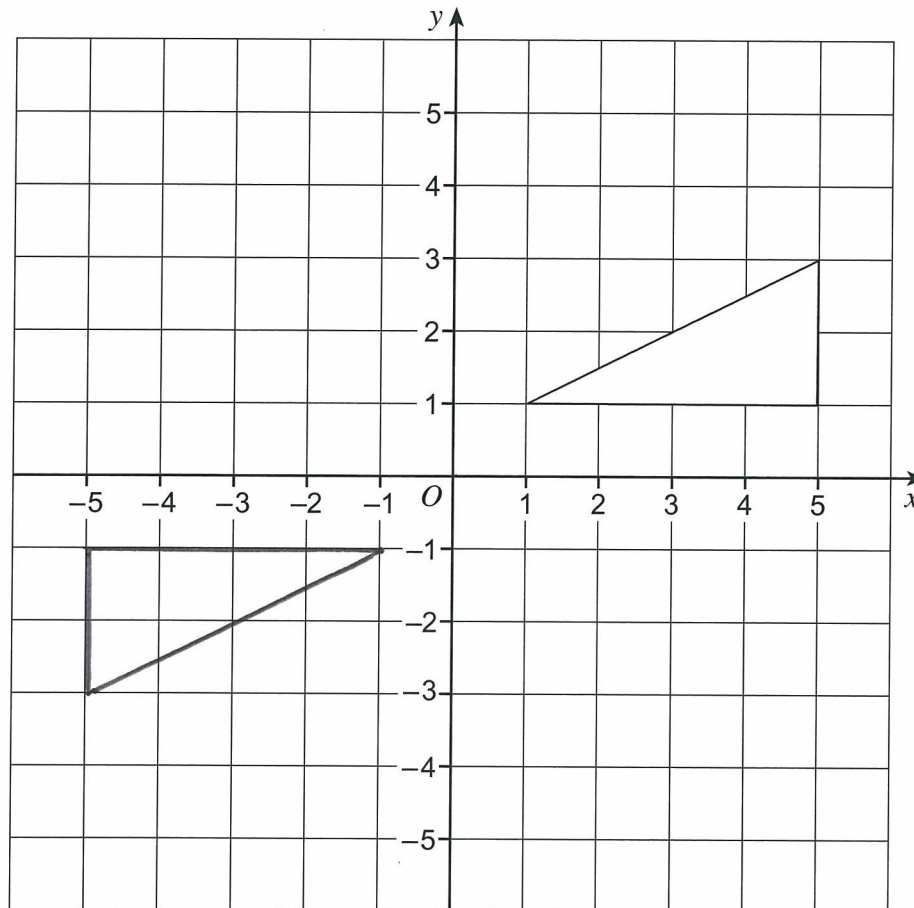
- 2 (a) Reflect the triangle in the line $x = 1$



(2 marks)



- 2 (b) Rotate the triangle through 180° about the origin.



(2 marks)

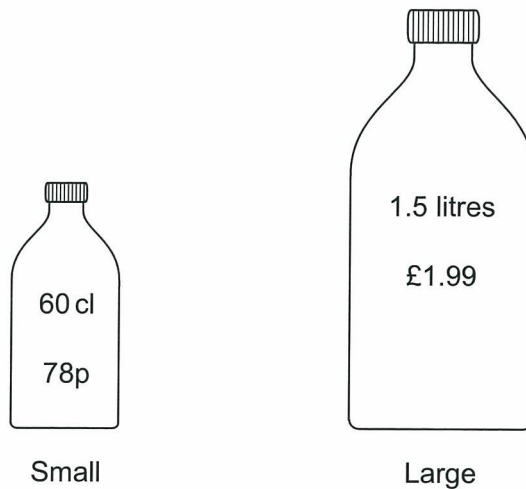
Turn over for the next question

Turn over ►



*3

The diagram shows two bottles of the same drink.



You are given that 1 litre = 100 cl

Which bottle is better value for money?

You **must** show your working.

$$1L = 1000ml \Rightarrow 1cl = \frac{1000}{100} = 10ml$$

$$60cl = 600ml \text{ and } 1.5L = 1500ml$$

Price per litre of small bottle is given by

$$\frac{100}{60} \times 78p = 130p \text{ and price per litre of large}$$

bottle is given by $\frac{199p}{1.5} = 132\frac{2}{3}p$

Answer Small bottle (4 marks)

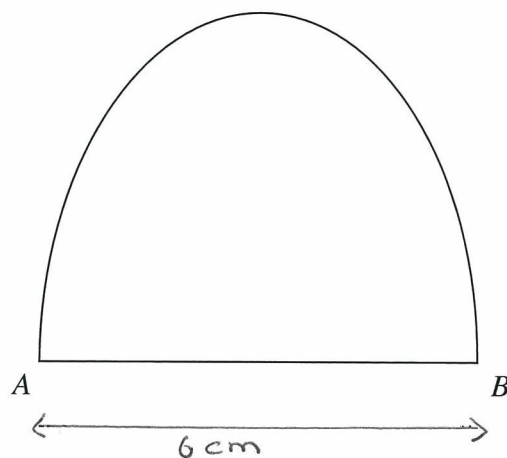
N.B: Even quicker would be to scale up the volume of the small bottle to the volume of the larger (or vice versa) and then compare the prices.

e.g. $\frac{1.5}{0.6} \times 78 = \frac{5}{2} \times 78 = 195p$ compared with the larger bottle @ 199p



4

Here is a scale drawing of a play area.



Scale 1 : 800

A straight wall is to be built from A to B.
250 bricks are needed for each metre of wall.

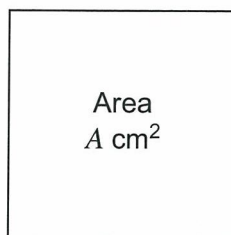
Work out the total number of bricks needed to build the wall.

$$\frac{6 \times 800}{100} \times 250 = 12,000 \text{ bricks}$$

Answer 12,000 (4 marks)



- 5 (a) The diagram shows a square piece of card.



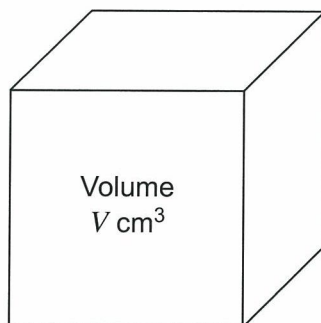
Width, w cm

Write down a formula connecting A and w .

.....

Answer $A = w^2$ (1 mark)

- 5 (b) This diagram shows a cube.



Width, w cm

Write down a formula connecting V and w .

.....

Answer $V = w^3$ (1 mark)



- 5 (c) The area of one face of a cube is 20 cm^2 .

Work out the volume of the cube.

For the cube above with length, l , width, w , and depth, d , $l = w = d = \sqrt{20} \text{ cm}$ and volume = $l \times w \times d = (\sqrt{20})^3 = 89.4 \text{ cm}^3$ (3 s.f.).

Answer 89.4 cm^3 (3 s.f.) (3 marks)

Turn over for the next question



- 6 (a) Three angles are in the ratio 2 : 3 : 7
The smallest angle is 60° .

Show that these three angles will fit together at a point with no gaps.

$2 : 3 : 7 = 60 : 90 : 210$
 i.e. the three angles in ascending order are
 60° , 90° and 210° . Since angles around a point
 add to 360° and $60^\circ + 90^\circ + 210^\circ = 360^\circ$, these three
 angles will fit together at a point with no gaps.

(3 marks)

- 6 (b) Two angles form a straight line.
One of the angles is $(x + 30)$ degrees.

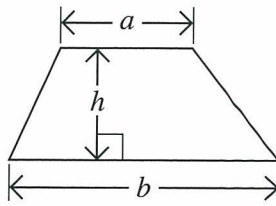
Write down an expression for the size of the other angle.
Give your answer in its simplest form.

$$180 - (x + 30) = 180 - x - 30 = 150 - x$$

Answer $150 - x$ degrees (2 marks)

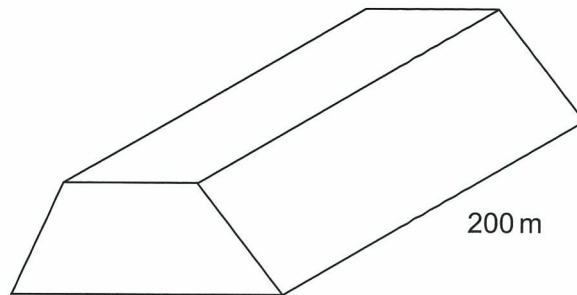


- 7 In the trapezium, $a = 6.5$ m, $b = 8.3$ m and $h = 3.2$ m



Not drawn accurately

The trapezium is the cross-section of a tunnel.
The tunnel is 200 metres long.



Work out the volume of the tunnel.

$$\begin{aligned}
 \text{Volume} &= \text{Cross-sectional area} \times \text{length} \\
 &= \frac{1}{2}(a+b)h \times l \\
 &= \frac{1}{2}(6.5 + 8.3)(3.2)(200) \\
 &= 4736 \text{ m}^3
 \end{aligned}$$

Answer 4736 m³ (4 marks)



8

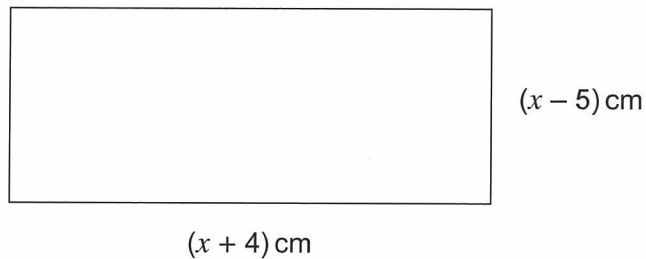
Solve the equation $x^2 - 5 = 0$
Give your answers to 1 decimal place.

$$x = \pm\sqrt{0+5} = \pm\sqrt{5} = \pm 2.2 \text{ (1 d.p.)}$$

Answer 2.2 and - 2.2 (2 marks)

9

The diagram shows a rectangle.



The area of the rectangle is 90 cm^2 .

Set up and solve a quadratic equation to work out the value of x .

$$(x+4)(x-5) = 90$$

$$\Rightarrow x^2 - x - 20 = 90$$

$$\Rightarrow x^2 - x - 110 = 0$$

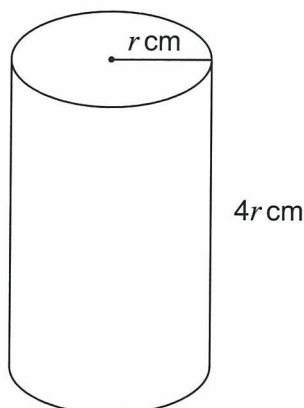
$$\Rightarrow (x - 11)(x + 10) = 0$$

$\Rightarrow x = 11$ or -10 of which we only require the positive solution for a spatial dimension.

$x = \dots\dots\dots 11 \dots\dots\dots \text{ cm}$ (5 marks)



- 10 The diagram shows a cylinder of radius r cm and height $4r$ cm.



- 10 (a) Work out a formula for the volume, V of the cylinder in terms of π and r . Simplify your answer.

$$V = \pi r^2 (4r) = 4\pi r^3$$

Answer $4\pi r^3$ (2 marks)

- 10 (b) Work out the volume of the cylinder when $r = 8$

$$V = 4\pi (8^3) = 6433.98 \text{ cm}^3 \text{ (2 d.p.)}$$

Answer $6433.98 \text{ cm}^3 \text{ (2 d.p.)}$ (2 marks)



11

This is a formula for the time to cook a turkey.

$$T = 15 + 20m$$

This is a formula for the time to cook a goose.

$$T = 40 + 15m$$

m is the mass in kilograms.

T is the time in minutes.

A turkey and a goose have the same mass and take the same time to cook.

Work out this time.

$$15 + 20m = 40 + 15m$$

$$\Rightarrow 5m = 25$$

$$\Rightarrow m = \frac{25}{5} = 5 \text{ kg.}$$

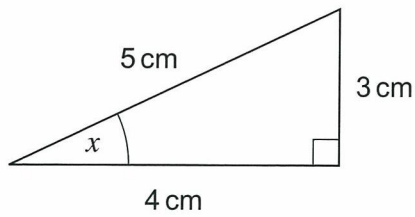
$$T = 15 + 20(5) = 15 + 100 = 115 \text{ minutes.}$$

$$\text{CHECK: } T = 40 + 15(5) = 40 + 75 = 115 \text{ mins.}$$

Answer 115 minutes (4 marks)



- 12 (a) The diagram shows a right-angled triangle.



Not drawn
accurately

Write down the value of $\sin x$.

Answer $\sin x = \frac{3}{5}$ or 0.6 (1 mark)

- 12 (b) In a different right-angled triangle, $\tan y = 0.7$

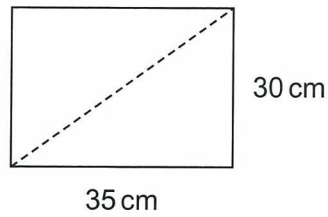
Work out the value of y .

Answer $y = \tan^{-1} 0.7 = 35.0^\circ$ degrees (3 s.f.) (1 mark)

Turn over for the next question



- 13 (a) The diagram shows a rectangle.



Not drawn accurately

Work out the length of the diagonal.

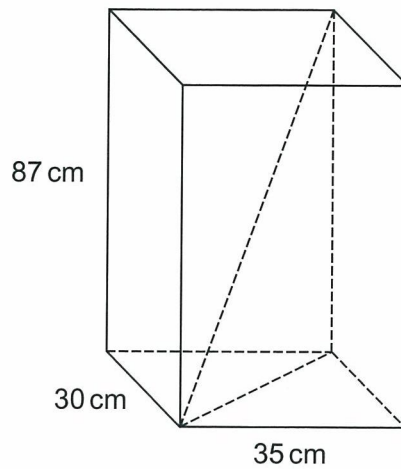
$$\sqrt{35^2 + 30^2} = \sqrt{2125} = 46.1 \text{ cm (3 s.f.)}$$

Answer 46.1 cm (3 s.f.) (3 marks)



13 (b)

The rectangle in part (a) is the base of this box.
The box is a cuboid.



Will a straight rod of length 1 metre fit in the box?
You **must** show your working.

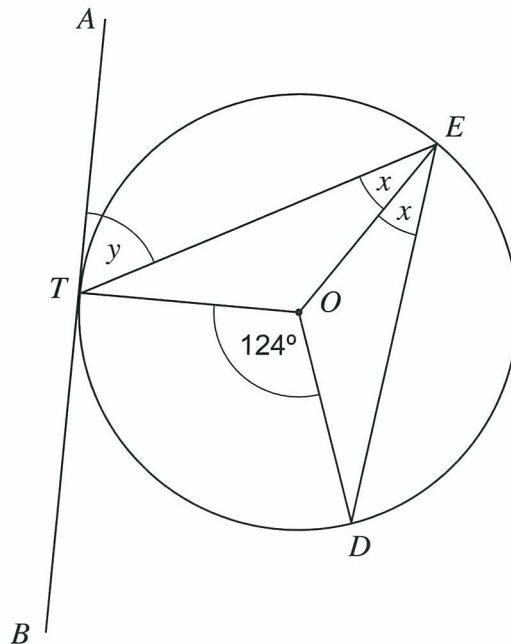
Let x = length of diagonal of cuboid.
 Then $x^2 = (\sqrt{2125})^2 + 87^2$
 $\Rightarrow x = \sqrt{9694} = 98.5 \text{ cm (3 s.f.)}$
 or 0.985 m (3 s.f.)
 \therefore A straight rod of length 1m WILL NOT
 fit in the box.
 (3 marks)

Turn over for the next question



14

The diagram shows a circle, centre O .
 ATB is a tangent at T .



Not drawn
accurately

14 (a)

Work out the value of x .

$$2x = \frac{1}{2}(124) = 62 \text{ as per 'The Double Angle Theorem'}$$

$$\Rightarrow x = \frac{62}{2} = 31^\circ$$

Answer 31 degrees (2 marks)

14 (b)

Work out the value of y .

$ATO = 90^\circ$ since a tangent and a radius form a right-angle where they meet.

$ETO = TEO = x = 31^\circ$ since base angles of an isosceles triangle are equal. $\therefore y = 90 - 31 = 59^\circ$

Answer 59 degrees (3 marks)



15

W is inversely proportional to x .
When $W = 6$, $x = 20$

Work out the value of W when $x = 24$

$$W \propto \frac{1}{x}$$

$$\Rightarrow W = \frac{k}{x}$$

$$6 = \frac{k}{20}$$

$$\Rightarrow k = 6 \times 20 = 120$$

$$\therefore W = \frac{120}{x} \text{ and when } x = 24, W = \frac{120}{24} = 5$$

Answer 5 (4 marks)

Turn over for the next question



- 16 (a) You are given that 1 mile = 1.6 kilometres

Convert $6\frac{1}{2}$ miles into kilometres.

$$6\frac{1}{2} = \frac{13}{2} \quad \text{and} \quad 1.6 = \frac{16}{10} = \frac{8}{5}$$

$$\frac{13}{2} \times \frac{8}{5} = \frac{104}{10} = 10.4 \text{ km}$$

Answer 104 km (2 marks)

- *16 (b) A manufacturer claims a car like mine uses 5.5 litres per 100 km.

My car does 50 miles per gallon.

Is my car using more or less fuel than the manufacturer claims?
You **must** show your working.

$$1 \text{ gallon} = 4.54609 \text{ litres}$$

$$50 \times 1.6 = 80 \text{ km}$$

Manufacturer's claimed consumption rate per
litre is given by $\frac{5.5}{100} = 0.055 \text{ L/km}$

$$\text{Actual consumption rate} = \frac{4.54609}{80} = 0.057 \text{ L/km (3d.p.)}$$

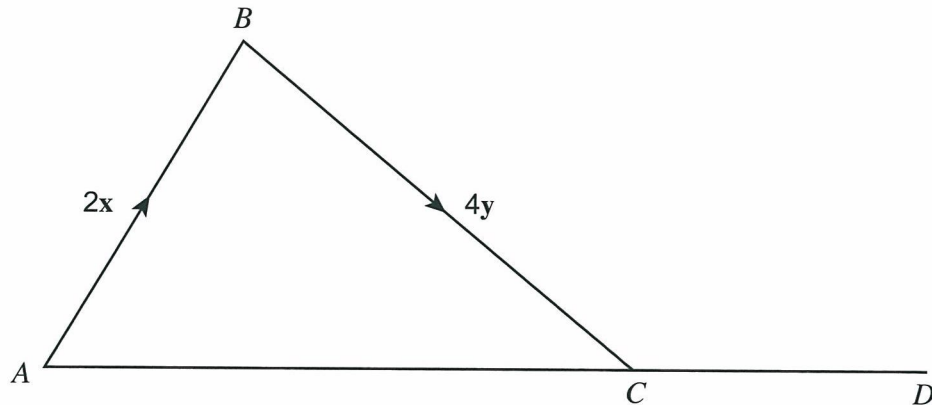
\therefore Car is using more Fuel than the
manufacturer claims.

(5 marks)



17

$\vec{AB} = 2\mathbf{x}$ and $\vec{BC} = 4\mathbf{y}$
 ACD is a straight line.



17 (a)

Write down the vector \vec{AC} in terms of \mathbf{x} and \mathbf{y} .

Answer $2\mathbf{x} + 4\mathbf{y}$ or $2(\mathbf{x} + 2\mathbf{y})$ (1 mark)

17 (b)

$AC : CD = 2 : 1$

Work out the vector \vec{AD} in terms of \mathbf{x} and \mathbf{y} .
 Give your answer as simply as possible.

$$\vec{AC} = \frac{2}{3} \vec{AD} \Rightarrow \vec{AD} = \frac{3}{2} \vec{AC} = \frac{3}{2} (2\mathbf{x} + 4\mathbf{y})$$

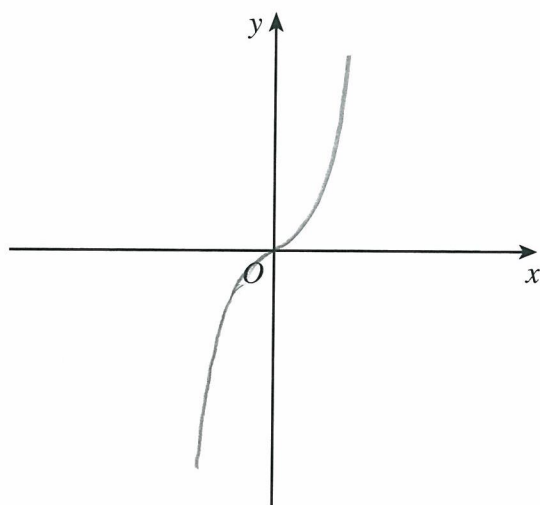
$$= 3\mathbf{x} + 6\mathbf{y} = 3(\mathbf{x} + 2\mathbf{y})$$

Answer $3(\mathbf{x} + 2\mathbf{y})$ (2 marks)

Turn over for the next question

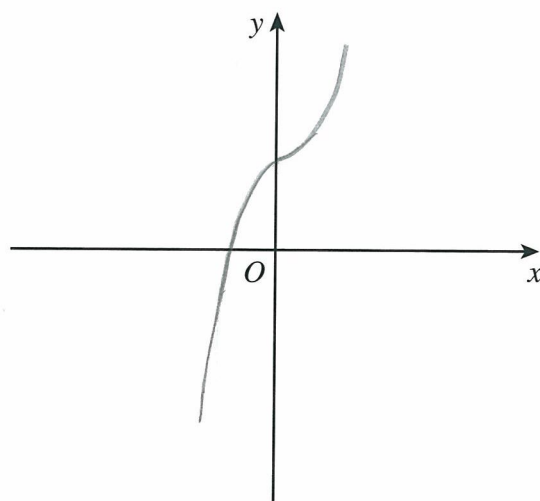


- 18 (a)** On the axes below sketch the graph of $y = x^3$



(1 mark)

- 18 (b)** On the axes below sketch the graph of $y = x^3 + 8$

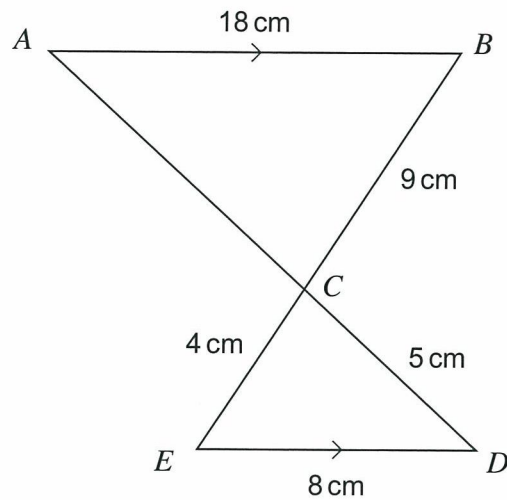


(1 mark)



19

ACD and BCE are straight lines.
Triangle ABC is similar to triangle DEC .
 AB is parallel to ED .



Not drawn
accurately

Work out the area of triangle ABC .

$$\frac{AC}{CD} = \frac{AB}{ED} \Rightarrow \frac{AC}{5} = \frac{18}{8} = \frac{9}{4}$$

$$\Rightarrow AC = \frac{9}{4} \times 5 = \frac{45}{4} = 11.25 \text{ cm.}$$

From the cosine rule, $11.25^2 = 18^2 + 9^2 - 2(18)(9)\cos x$
where x is the angle ABC .

$$x = \cos^{-1} \left[\frac{18^2 + 9^2 - 11.25^2}{2(18)(9)} \right] = 30.8^\circ \text{ (3 s.f.)}$$

$$\therefore \text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} (18)(9) \sin x = 41.4 \text{ cm}^2 \text{ (3 s.f.)}$$

Answer 41.4 cm^2 (3 s.f.) (6 marks)

END OF QUESTIONS



There are no questions printed on this page

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